# Modelling of precipitation extremes in a transient Regional Climate Model run for the Rhine basin


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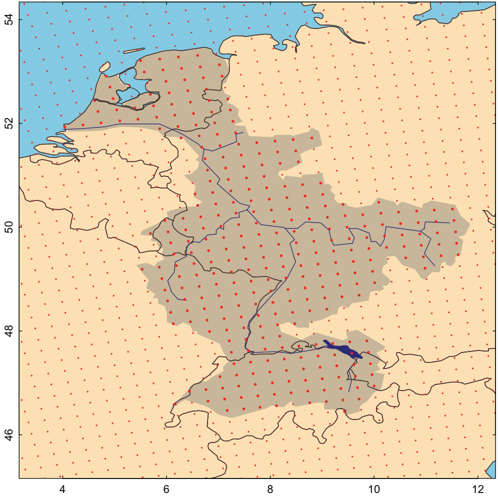
ENSEMBLES WP5.4


De Bilt, 14th May 2008

# The Rhine basin




- area $\doteq$ 185.000 km$^2$
- RACMO_ECHAM5 RCM, gridsize = 25 x 25 km
- transient run (1950-2100) under SRES A1B scenario
- modelling of
  1 day summer (JJA) and
  5 day winter (DJF)
  precipitation extremes

# GEV distribution function

$$F(x) = \exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}, \qquad \xi \neq 0$$

$$F(x) = \exp\left\{-\exp\left[-\left(\frac{x-\mu}{\sigma}\right)\right]\right\}, \qquad \xi = 0$$

- $\mu$ ... location parameter
- $\sigma$ ... scale parameter
- $\xi$ ... shape parameter

# GEV distribution function

GEV parameters $\mu$, $\sigma$, and $\xi$

- may vary over the region (spatial heterogeneity)
- may vary over time (climate change)

A popular assumption about spatial heterogeneity is that

- $\mu$ varies over the region
- $\xi$ and $\gamma = \frac{\sigma}{\mu}$ are constant over the region

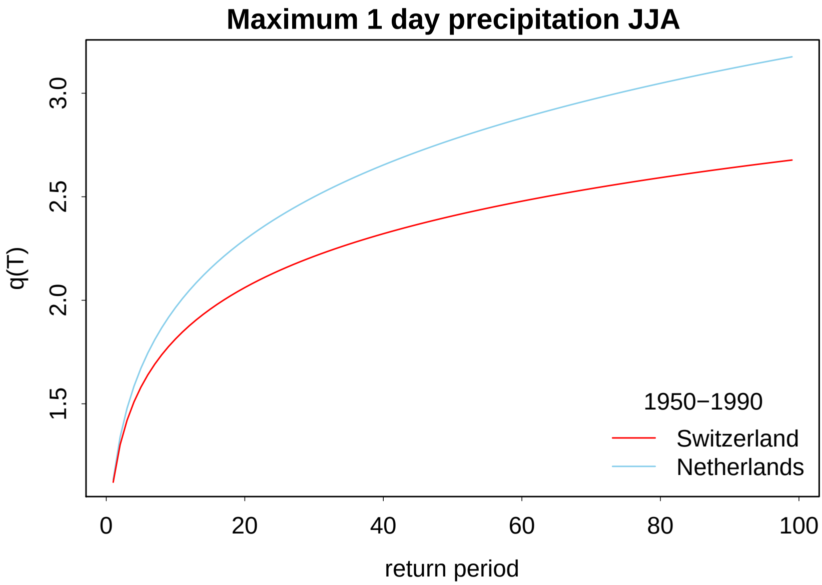
  ($\gamma$ is a dispersion coefficient comparable with the coefficient of variation)

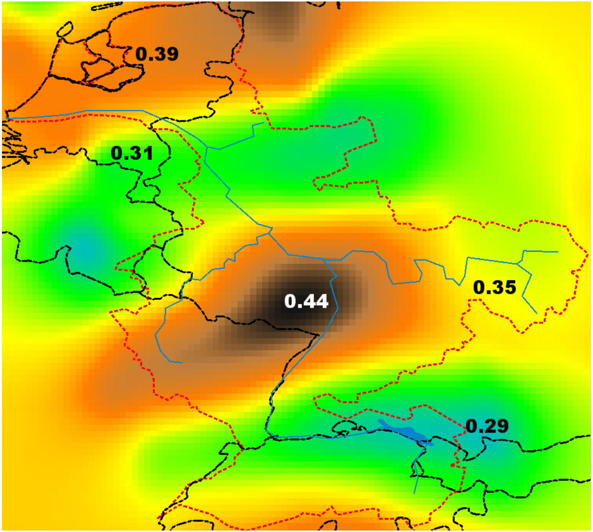
$\Rightarrow$ The $T$-year quantile $X(T)$ at any location can be represented as

$$X(T) = \mu \cdot q(T)$$

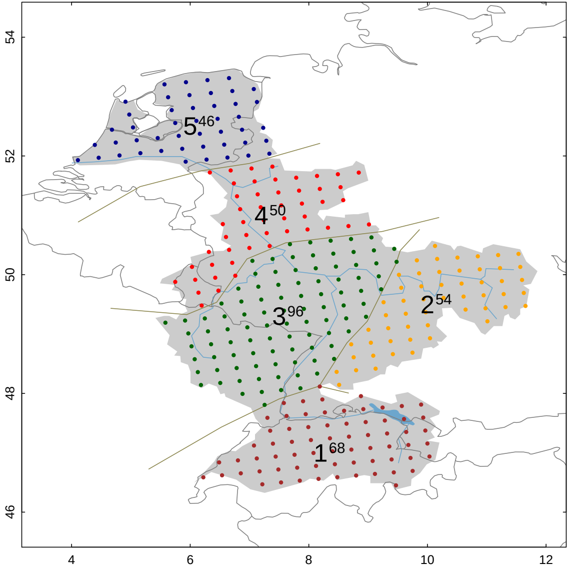
where $q(T)$ is a common dimensionless quantile function (growth curve):

$$q(T) = 1 - \gamma \cdot \frac{1 - \left[-\log\left(1 - \frac{1}{T}\right)\right]^{-\xi}}{\xi}$$


Maximum 1 day precipitation JJA
q(T)
3.0
2.5
2.0
1.5
0
20
40
60
80
100
return period
1950−1990
Switzerland
Netherlands




Maximum 1 day precipitation in JJA (1950-1990)


5 46
4 50
3 96
2 54
1 68
54
52
50
48
46
4
6
8
10
12

# Model used

- location parameter varies over the area, but with common trend

$$\mu(s,t) = \mu_0(s) \cdot \exp\left[\mu_1 \cdot (t - t_0)_+\right]$$

($s$ is grid box indicator, $t$ is year, $t_0$ is start of the trend)

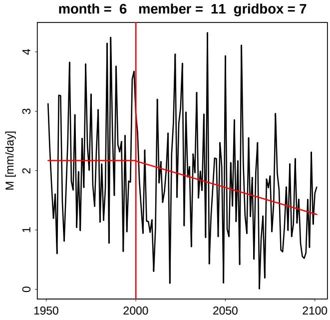
- dispersion coefficient and shape parameter are constant over the area, but vary with time

$$\gamma(t) = \gamma_0 \cdot \exp\left[\gamma_1 \cdot (t - t_0)_+\right]$$

$$\xi(t) = \xi_0 + \xi_1 \cdot (t - t_0)_+$$

- the parameters are estimated by maximum likelihood

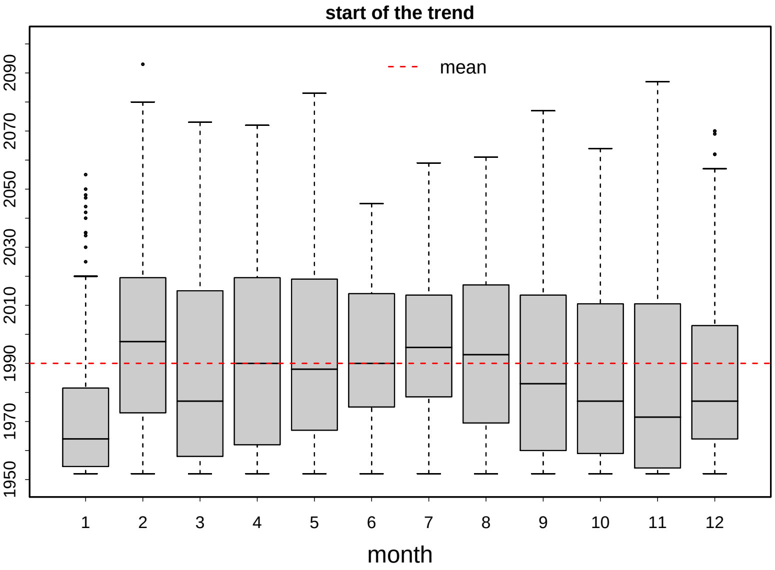
For the determination of $t_0$ the Essence project data were used.



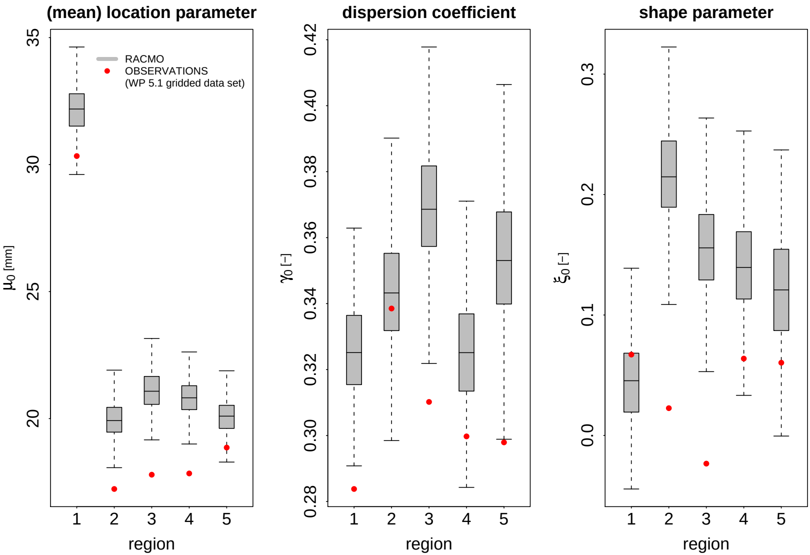

- ECHAM5/MPI-OM global model
- 17 ensemble members were analyzed at 8 gridboxes
- the model

$$M_t = a + b \cdot (t - t_0)_+$$

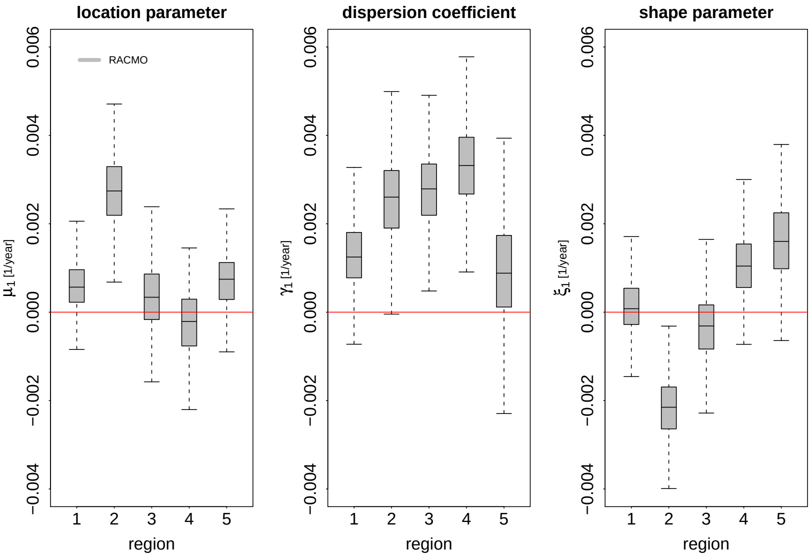
was fitted for each month, each gridbox and each ensemble member ($M_t$ is monthly precipitation in mm/day)


start of the trend
mean
1950
1970
1990
2010
2030
2050
2070
2090
1
2
3
4
5
6
7
8
9
10
11
12
month

# Resulting parameters (JJA)



# Resulting parameters (JJA)



Reduction [%] in standard errors due to spatial pooling

| parameter | region 1 | 2 | 3 | 4 | 5 | average |
|---|---|---|---|---|---|---|
| $\mu_1$ | 37 | 32 | 34 | 31 | 39 | 35 |
| $\gamma_0$ | 58 | 45 | 48 | 53 | 44 | 50 |
| $\gamma_1$ | 67 | 60 | 60 | 61 | 58 | 61 |
| $\xi_0$ | 73 | 71 | 67 | 66 | 58 | 67 |
| $\xi_1$ | 80 | 75 | 75 | 72 | 66 | 74 |

- $T$-year quantile at time $t$ and location $s$:

$$Q(T,s,t) = \mu(s,t) \cdot q(T,t)$$

- Relative change between $t_2$ and $t_1$ $(t_2 > t_1 \geq t_0)$ is:

$$\frac{Q(T,s,t_2)}{Q(T,s,t_1)} = \frac{\mu(s,t_2)}{\mu(s,t_1)} \cdot \frac{q(T,t_2)}{q(T,t_1)}$$

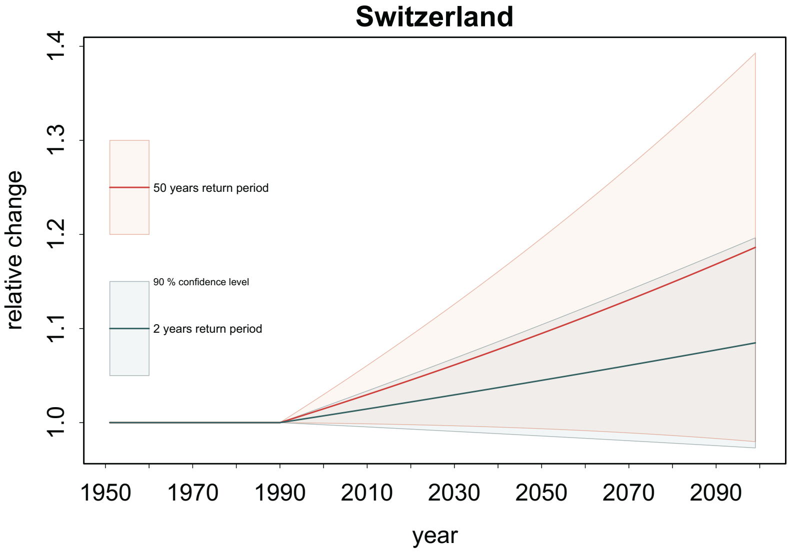
$$\mu(s,t) = \mu_0(s) \cdot \exp\left[\mu_1 \cdot (t - t_0)_+\right] \Rightarrow \frac{\mu(s,t_2)}{\mu(s,t_1)} = \exp\left[\mu_1 \cdot (t_2 - t_1)\right]$$

$\Rightarrow$ the relative change in quantiles can be written as

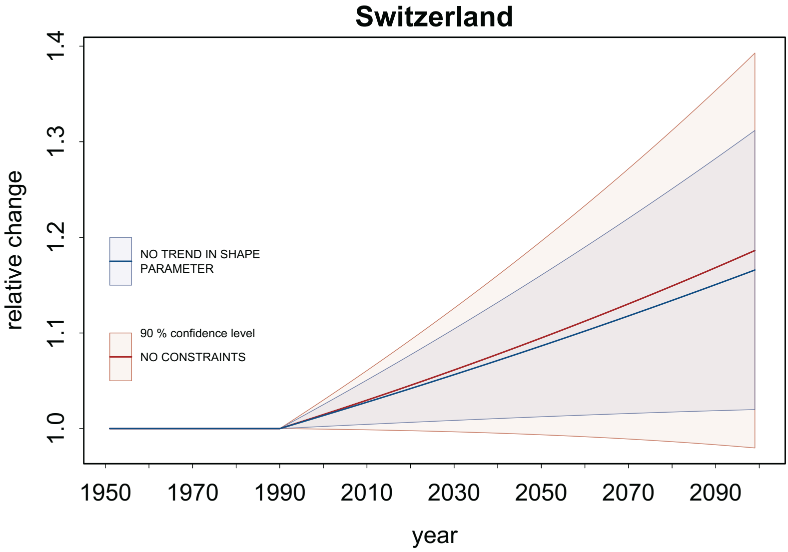
$$\frac{Q(T,s,t_2)}{Q(T,s,t_1)} = \exp\left[\mu_1 \cdot (t_2 - t_1)\right] \cdot \frac{q(T,t_2)}{q(T,t_1)}$$

which does not depend on $s$.

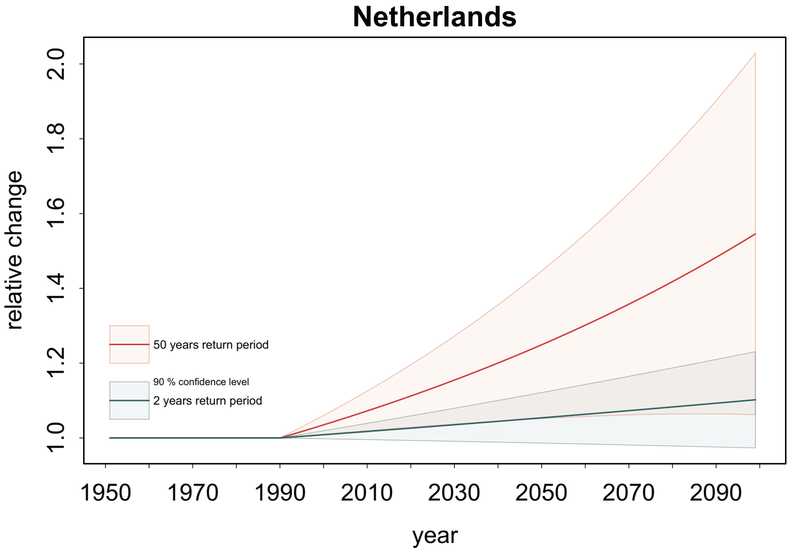
# Relative change in quantiles (JJA)



# Relative change in 50-year quantile (JJA)



# Relative change in quantiles (JJA)



# Model diagnostics

For each grid box we calculate the residuals:

- we transform the seasonal maxima $X_t$ to:

$$\widetilde{X}_t = \frac{1}{\hat{\xi}(t)} \cdot \log\left[1 + \frac{\hat{\xi}(t)}{\hat{\gamma}(t)} \cdot \left(\frac{X_t}{\hat{\mu}(t)} - 1\right)\right],$$

  which should have a standard Gumbel distribution;

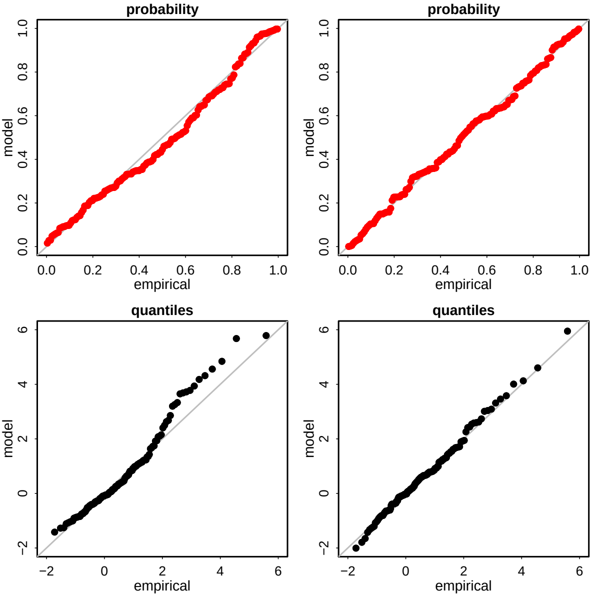
$$\Pr\{\widetilde{X}_t \leq x\} = \exp\left[-\exp(-x)\right]$$

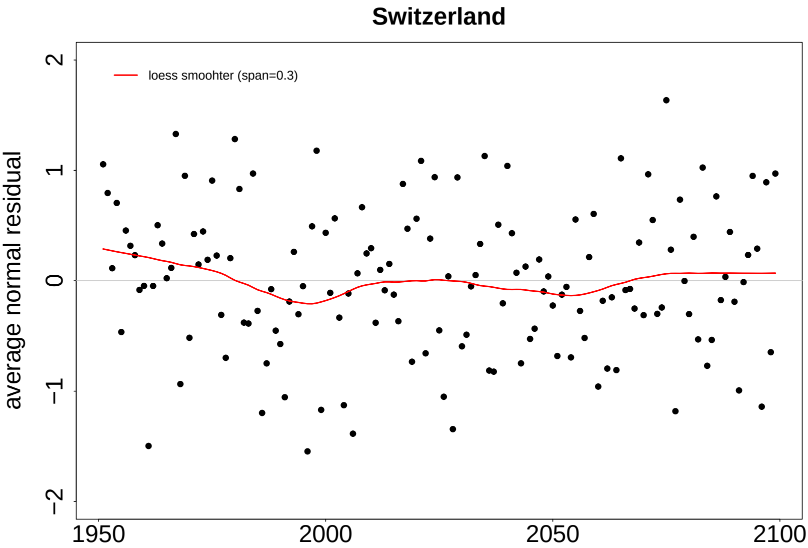
  if the model is true.

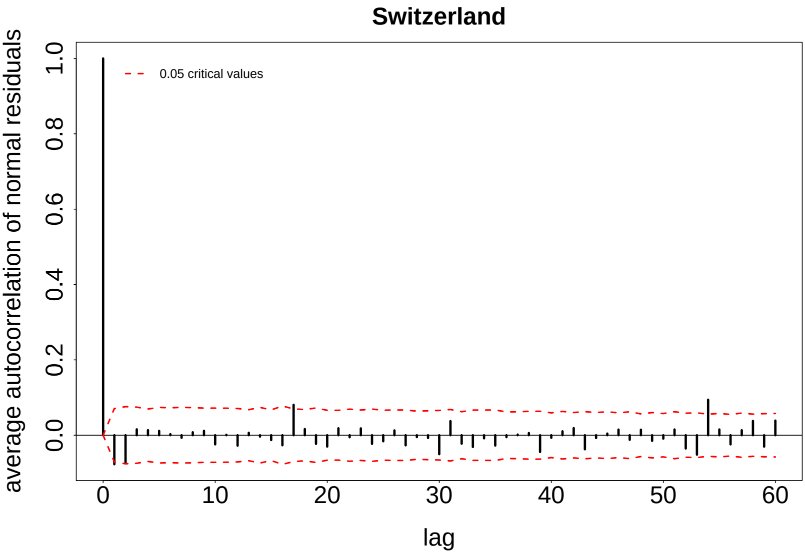
- $\widetilde{X}_t$ can be further transformed to have a standard normal distribution:

$$\text{normal residuals:} \quad \Phi^{-1}\left\{\exp[-\exp(-\widetilde{X}_t)]\right\} \sim \mathrm{N}(0,1)$$

# Model diagnostics




Switzerland
loess smoohter (span=0.3)
average normal residual
2
1
0
−1
−2
1950
2000
2050
2100


Switzerland
0.05 critical values
average autocorrelation of normal residuals
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
60
lag

- Draw a sample from the years 1950, ..., 2099:

$$j_1, \ldots, j_t, \ldots, j_{150}$$

- Take for each resampled year the vector of standardized maxima:

$$\left(\widetilde{X}_{j_t}(1), \ldots, \widetilde{X}_{j_t}(s), \ldots, \widetilde{X}_{j_t}(S)\right),$$

  with $S$ the number of grid boxes.

- Transform this vector back to the original scale:

$$X_t^*(s) = \hat{\mu}(s,t) \cdot \left\{ 1 + \hat{\gamma}(t) \cdot \frac{\exp\left[\hat{\xi}(t) \cdot \widetilde{X}_{j_t}(s)\right] - 1}{\hat{\xi}(t)} \right\}, \quad s = 1, \ldots, S.$$

# Conclusions

- Regional GEV modelling provides an informative summary of changes in parameters and various quantiles (rather than a single quantile only).
- Taking $\xi$ and $\gamma$ constant over a region strongly reduces standard errors.
- However, there is still much uncertainty about the change in large quantiles, in particular in the case of a non-homogeneous change of the shape parameter.
- Joint significance of lack of fit tests requires further research.