An acoustical array for subsonic signals

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AN ACOUSTICAL ARRAY FOR SUBSONIC SIGNALS

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Abstract
Array technique well known in seismology is applied to acoustical signals in the subsonic or infrasonic frequency band. This is achieved by simply changing the detector, and scaling the dimensions of the array according to the differences between acoustic and seismic velocities. The design of a robust detector is taken from Hugo Benioff. The atmosphere is more homogeneous than the solid earth on length scales of the array but varies strongly with time due to wind and temperature variations. This leads us to reevaluate the array design. Recent techniques to process the data, borrowed from seismology, are described. Finally various trade-offs in array processing are mentioned in order to make clear that different objectives could guide between various array processing techniques. Some results are discussed from measurements with a six-element array mainly focussed at sonic boom detections.

Introduction
The application of array techniques is well established in seismology. The measurement of coherent seismic waves by means of a collection of separate detectors, each having equal properties, yields valuable information such as horizontal wave velocity, azimuth of the incoming wave and dispersion properties. These are virtually all the parameters of a given plane-wave field in space and time. By summing the individual time series of the detectors with appropriate time delays, some noise reduction can be achieved. Usually this reduction in noise is of the order of $\sqrt{N}$, where $N$ is the number of sensors. In the field of acoustics the concept of an array of detectors seems to be less common than in seismology. Although in the nineteen-sixties arrays of microbarographs were employed to detect pressure waves from atmospheric nuclear explosions: Large Aperture Microbarograph Array (LAMA). For the same reason today the interest is growing in the context of monitoring a comprehensive nuclear testban.
In the late sixties a number of detectors were developed also with the purpose to detect nuclear explosions: low frequency microphones, the Daniels Pipe arrays (Cook, 1971), microbarographs etc. One of the detectors which we employed in this study dates from before this period. It was constructed by Hugo Benioff in 1939. The detector consists of a low frequency loudspeaker mounted in a closed airtight volume, and it is operated as a pressure detector. The response is flat to pressure change. Surprisingly another low frequency detector is available that is both very small and low budget: an electret microphone. This response is flat to pressure.

Current developments both in computers and in mechanical technology make that today highly efficient and low cost acoustic arrays are possible. Also when equipped with more sophisticated pressure sensors such as very low frequency quarts crystal pressure sensors.

Not only the ideas on the instrumentation adapted were from seismology. Also, the motivation of this study had a seismological background. In the northern part of the Netherlands up till 1991 seven earthquakes occurred that are induced by natural gas extraction. Besides these well-recorded events we received numerous reports of rumblings and vibrations. Sometimes these reports were covering an area of over 10,000 km², sometimes only reports were given from a very limited area. From analysis of the Netherlands network of seismographs it followed that these events were all of atmospheric origin; low frequency sound waves in the seismic frequency band.

Further analysis of these signals seems appropriate. The occurrence of other historic mysterious sounds strengthen us to investigate these findings. From 1900 onwards people along the coast of the Netherlands experienced the so-called "zeepoeffers" most often in the summer with calm weather conditions. These brontides (from the Italian brontidi), as these sounds are called in the Anglo-saksian literature (Gold and Soter, 1979) are often observed in coastal regions without being explained by a common physical phenomenon. To our surprise the signals in the low audible and infrasonic frequency range provided a model example of the application of array techniques.
Instrumentation

The instrumentation of the acoustical array we designed consists of a set of six acoustical detectors and the central data acquisition computer including amplifiers, anti-alias filters and a time-keeping system. We accomplished the off-line analysis with a separate personal computer. Data transfer between the data-acquisition PC and the off-line analysis PC was done via floppy disks or via a local area network. For remote sites we used dialup telephone lines with the PC-Anywhere program, between PC’s we used the Lantastic filetransfer program. Finally the data is stored on an optical disk. In figure 1 an overview is given of the whole system.

The acoustic detector

The design of the low frequency microphone we choose is dating from 1939 (Benioff, 1939). B. Gutenberg used this kind of loudspeaker-microphone to probe the velocity structure of the atmosphere (Gutenberg, 1939). It consists of a low frequency loudspeaker (woofer) mounted in a box, just like a pressure box in audio technique. When the atmospheric pressure outside the box is changing, the cone of the speaker will move like a piston. The magnet-coil assembly will generate a voltage proportional to the velocity of the cone. Usually a small leak is introduced to equalize the pressures over long time periods, such as from temperature variations and changing ambient atmosphere pressure. The movement of the cone \( \Delta x \) upon a pressure variation \( \Delta P \) is given by:

\[
\Delta x = \Delta P \frac{-A}{A^2 \gamma P/V + C_e}
\]  

(1)

In which \( A \) is the area of the cone, \( P \) is the ambient atmospheric pressure, \( V \) is the enclosed volume of the box, \( \gamma \) is the ratio of heat capacities at constant pressure and constant temperature. \( C_e \) is the force constant of the elastic hinges of the cone itself. This force constant can be calculated from the resonance frequency \( f_{res} \) of the unmounted speaker:

\[
f_{res} = \frac{1}{2 \pi} \sqrt{\frac{C_e}{m_e}}
\]  

(2)
The mass of the cone is $m_c$. The output voltage $V_{out}$ of the detector is proportional to the velocity of the cone:

$$V_{out} = K \frac{d(\Delta X)}{dt}$$  \hspace{1cm} (3)

$K$ is the generator constant of the magnet-coil combination. The output voltage is a measure of the pressure change. From equation (1) it becomes clear that if a speaker with a very low resonance frequency is used, so $C_e$ is small compared to $A^2 \gamma P/V$, the system sensitivity is given by:

$$\frac{\Delta X}{\Delta P} = \frac{-V}{A \gamma P}$$  \hspace{1cm} (4)

So one can optimize the output by enlarging the enclosed volume $V$ while keeping a small diameter of the loudspeaker. This is the case of a true piston. When the opposite is true, $C_e$ is large compared to $A^2 \gamma P/V$ then the system’s sensitivity is given by:

$$\frac{\Delta X}{\Delta P} = \frac{A}{C_e}$$  \hspace{1cm} (5)

In this case a larger area of the speaker will increase the sensitivity. This is the case of a pure elastic bellows. The trade-off between $V$, $C_e$, and $A$ can be found by differentiating equation (1) with respect to $A$, in order to maximize the sensitivity:

$$A = \sqrt{\frac{C_e V}{\gamma P}}$$  \hspace{1cm} (6)

Most of the above formulas also apply to microphones with elastic membranes or bellows. With slight modifications the same holds when feedback is applied. In that case the membrane is driven back to the equilibrium position with an external force that balances the external pressure. This stiffens the elastic system and enlarging $C_e$. Feedback transfers the dynamics of the system from the mechanical parts to the electronics, where it is easier to handle.
When the detectors are placed in the open field several factors are important to ensure stable operation with respect to wind and weather. Figure 2 shows most of the precaution measures. The system is placed upside down to avoid rain coming in. In order to avoid large pressure variation due to wind the detector is placed close to the ground on a lawn and Dacron wool (fibre fill) is used. After one year of operation we found that this material was disappearing in spring because it was considered most useful by birds.

The detectors we used had a volume of 30 litre and were equipped with the Philips speaker AD10202/W8. The choice of this speaker was mainly motivated by the availability of accurate data sheets: notably the parameters $K, A, C \varepsilon$ and $f_{ms}$. We noticed that with very low ambient temperatures (−20 °C) the elastic rubber ring around the stiff cone lost its elastic properties. In this case the output levels were reduced.

The dynamical response of the detector is given in figure 3. Clearly there are 3 regimes, the low frequency regime: the small leak is levelling out the long period pressure differences. The time constant of the leak determines the corner frequency. The middle frequency range is the working range of the detector. The high frequency range, beyond the resonance frequency of the speaker is a second order roll-off towards higher frequencies. Considering the dynamic response of the detector, it is efficient to choose the resonance frequency of the mounted loudspeaker close to the Nyquist frequency of the data-acquisition system or the corner frequency of the anti-alias filter.

Besides the 30-litre detector we experimented with a 5-litre version. The much smaller detector (30 x 15 cm) works equally well provided a low noise preamplifier to compensate for the ten times lower output voltage. The speaker we used was a woofer from a car stereo system with smaller dimensions (VISATON WS 13 NG, 8 Ohm). Still smaller dimensions are possible. One of the advantages of this kind of detector is its inherent low-price and easy construction. Some care should be taken to ensure that the controlled leak is not bypassed by the possible leak of the porous material of the cone of the speaker.
The electret-detector

Quite by accident we discovered that cheap and small electret microphones have reasonable flat responses down to 1 Hz. In comparison with the loudspeaker detector the electret microphone is generally more sensitive towards lower frequencies. Within a given set of electret microphones a slight variation occurs of the low-frequency response and output level, therefore we selected approximate equal individuals from a larger set. We operated acoustic arrays with both (woofer and electret) detector types. When infrasonic waves are measured with this microphone, the highest noise amplitudes are due to wind induced pressure variations. Therefore, considerable care should be taken to suppress this noise.

Noise reducing structures

We used spatial filtering to reduce the noise signals, since we wanted to keep the flat frequency response. Acoustic signals are not attenuated considerably when integrated over areas smaller than half their wavelength. In general, due to the temperature profile as a function of height the acoustic waves propagate almost in a horizontal direction over the array. This is a marked difference with seismic wave propagation. The acoustic wavelength corresponds rigidly to a certain frequency, only depending on the velocity of sound. Due to the turbulent character of wind field, pressure variations at a certain frequency correspond on average to a variety of wavelengths. Therefore, one can use spatial filtering such as spatial integration to discriminate between acoustic and wind signals.

Daniels (Cook 1971) solved this problem earlier for very low frequencies by using large circular pipe arrays. We designed a sensor in the frequency range from 1 Hz to 50 Hz by scaling down the circular pipe. A porous garden hose (Gardena) works equally well. The electret microphone is mounted inside a thick walled (3.5 mm) plastic tube. The outside diameter is 32 mm. The 8-metre tube is bend into a closed circular shape with a diameter of 2.5 metres. The outside pressure is measured by the microphone through twenty equidistant holes 0.8 mm in diameter.
The acoustic resistance of the holes is far higher than the resistance of the tube itself. Therefore, the pressure inside the tube is the spatial average of the ring's circumference. The whole ring is buried below 5 to 10 cm course gravel, to avoid direct impact of the wind. In comparison with a single point detector a tenfold reduction of wind noise was achieved with the ring-detector. See figure 4. Another possibility is a noise reducing structure that has radial tubes. In the centre of the “spider” the detector is situated. The spacing of the small holes can than be subject to optimalisation. When the number of radial tubes exceeds six, the circular tube is more material efficient.

A still somewhat an unsolved problem is the accumulation of moist. We have tested various solutions none with satisfactory results. The best solution is probable to heat the sensor at temperatures slightly above the ambient temperature. The problem is most pronounced with the electret microphones. Another problem arrises when small particles and dirt are clogging the small holes in the noise reducing pipe. These practical issues require further research.

**The data-acquisition system**

The PC-based data-acquisition system was built with standard boards. With the 16 channel amplifiers and multiplexer board, the six acoustic detectors were recorded with two different gains. Also, a separate time channel was included consisting of a DCF77 radio clock. This clock made an absolute time calibration possible. We used a 12-bit A/D converter of the type DT 2824 or 2821 of Data Translation. We also used a 16-bit A/D converter of the type PCL-816/814B of PC-LabCard from Advantech Co., Ltd. with an adjusted data-acquisition program. The sampling frequency was set at 120 Hz. The backbone of the data-acquisition system was software from the IASPEI Toolbox, Volume I (Lee, 1989). This software is in standard use by many seismological stations. We altered the program to accommodate the 16-bit A/D converter, to incorporate filtering procedures and to list a number of auxiliary parameters on-line, such as the temperature.
A critical part of the software is the event detector. On each channel a long term average over short term average trigger (LTA/STA) is run. Only when five out of six channels are triggered an event is declared and the data is stored on disk. This double event trigger works quite well. Even in stormy weather the number of false events stayed within acceptable numbers. However, it may take some time to adjust the parameters of the LTA/STA detector. We intend to install just like our seismic systems a large (1 Gbyte) harddisc in the data acquisition PC’s. Such a data storage device is a large ringbuffer. In this way we are less dependent on the trigger algorithm.

The acoustic array

A collection of separate detectors with similar properties is called an array. The detectors of an array sample signals in the spatial domain. The coherent signals from the detectors are supposed to originate from plane waves in most cases. Sampling in the time domain is usually accomplished with equidistant sample time intervals. Since in the spatial domain each sample is the equivalent of a detector there is the need for a much more efficient design of an array.

With an array one measures essentially the spatial form of the incoming waves. After the Fourier transforms involving space and time one obtains the frequency-wavenumber spectrum. Consequently the phase velocity in the horizontal plane at each wavelength or frequency of the wave train is known. When also the temperature is measured the phase velocity of the wave train is known.

In many problems the velocity of plane waves is not the most practical unit, since the physical plane wave quantities involved propagate with a functional dependence on space and time via:

\[ (t - \frac{\hat{t} \cdot \vec{r}}{c}) = (t - \hat{p} \cdot \vec{r}) \]  \hspace{1cm} (7)

The unitvector \( \hat{t} \) is the direction of propagation the normal of the wave front, \( t \) and \( \vec{r} \) are the time and spatial coordinates respectively, and \( c \) is the wave propagation velocity or phase velocity.
The vector:

$$\vec{p} = \frac{\hat{t}}{c}$$

(8)

is called slowness vector. The slowness, e.g. the reciprocal of the apparent velocity in a certain direction with unit direction vector \( \hat{r} \) is given by the product \( \vec{p} \cdot \hat{r} \). Since most arrays are situated on the earth's surface, the horizontal component of slowness is measured.

The relation between horizontal wavenumber \( \vec{k}_h \), horizontal slowness \( \vec{p}_h \) and frequency \( \omega \) is given by:

$$\vec{k}_h = \omega \vec{p}_h$$

(9)

For sound waves with very low dispersion it is most handy to produce plots of the power \( P \) as a function of the slownesses \( p_x \) and \( p_y \) instead of the wavenumbers \( k_x \) and \( k_y \). This is called a the slowness plot. An easy, but not necessarily the best way to produce such plots is via the expression for the computation of frequency-slowness spectra:

$$P(\omega, \vec{p}_h) = \left| \sum_{n=1}^{N} S_n(\omega) e^{i \omega p \cdot \vec{r}_n} \right|^2$$

(10)

The complex Fourier spectrum of the \( n^{th} \) detector is \( S_n(\omega) \), \( \vec{r}_n \) is the position vector of the \( n^{th} \) detector. \( N \) is the total number of detectors. This calculation is usually followed by an integration over a certain frequencyband \( \omega_j, \omega_2 \):

$$P(\vec{p}_h)_{\omega_j, \omega_2} = \int_{\omega_j}^{\omega_2} P(\omega, \vec{p}_h) \, d\omega$$

(11)

The integration over the frequencyband can in principle be omitted when narrow band signals of known frequency are considered. With broadband signals however, this integration is very effective to lower unwanted peaks in the power plot \( P \). The main lobe is then more easy to identify.
Array design

Since a finite number of spatial sampling points produce a distortion in the measured two-dimensional wavenumber or slowness spectra, it is of prime importance to optimize the array design. The distortion arises because the measured spectrum is a convolution of the true spectrum with the array response function. The array response function is given by:

$$R(\omega, p) = \frac{1}{N} \sum_{n=1}^{N} e^{i\omega \cdot \mathbf{r}_n}$$

(12)

Usually plots are presented of $RR^*$ or $|R|$ as a function of $p$, and $p_j$ for a specific frequency $\omega$, sometimes the power is given in dB, so a logarithmic scale is used.

In the following discussion on array design we give the arguments given by Haubrich (Haubrich, 1968). In general we want to optimize the array response with respect to three features:

a. the resolution of the main central lobe (large resolution),

b. the distance between the central lobe and the other (alias) main lobes (large Nyquist wavenumber, bandwidth),

c. the amplitude ratio between the central lobe and the secondary lobes, e.g. between the central lobe and (alias) main lobes (low ripple between main lobes, errors).

To reach a trade-off between these conflicting demands on the array response we have three distinct categories of parameters available to obtain optimal power spectra. It should be noted however that “optimal” usually strongly depends of the specific problem at hand. The categories are:

a. the $N$ coordinates $\mathbf{r}_n$ of the detector sites,

b. the weights applied as a tapering function either as a function of the relative position of the detectors or as a function of the various detector separation distances,

c. the specific method of array processing techniques, subsequent data analysis and power spectrum estimation, e.g. high resolution methods, the use of a cleaning procedure etc.
This problem of optimization cannot be solved with general methods. However, a reasonable way to approach this problem is first to find a convenient array lay out (a) by trial and error methods. Then solve analytically the weight problem (b). The subsequent choice of an array processing technique (c) depends usually strongly on the specific boundary conditions for the problem at hand. When classical methods (low resolution) are chosen the demands on the resolution are normally solved through the choice of the number of array elements and the array lay out.

It is noted however, that to optimize an array the $N$ time series themselves should be explicit part of the procedure and not just a simple one frequency sinusoidal signal. This opens the possibility for efficient correlation techniques and is a way around to strict wavenumber bounds. In general larger arrays are possible without aliasing problems. It is also noted that the array response is known with precision. It is therefore possible to deconvolute the measured f-k spectra.

If we work out the expression $RR^*$ for the array response and introduce a weight function $w_{js}$, we obtain the expression:

$$RR^* = \sum_{j=1}^{N} \sum_{s=1}^{N} w_{js} e^{i\omega \Phi(r_j - r_s)}$$

(13)

This expression follows also when the power spectrum is estimated from the Fourier transform of the cross-correlation (Smart, 1971).

The array with the sample points $r_j$ has an associated set of points defined by:

$$\bar{D}_m = \bar{r}_j - \bar{r}_s$$

(14)

This set is called the coarray, it consists of $N^2$ points of which $N$ are zero. From the definition is it clear that the points $\bar{D}_m$ occur in pairs: $\bar{r}_j - \bar{r}_s, \bar{r}_s - \bar{r}_j$. 
There are at most $\hat{M} = N(N-1)/2$ pairs. In symmetrical arrays some $\vec{r}_i - \vec{r}_j$ values produce the same points in the coarray. We call the number of independent pairs of points of the coarray $M$. With this, the expression (13) can be rewritten as:

$$RR^* = w_o + 2 \sum_{m=1}^{M} w_m \cos(\omega \vec{p} \cdot \vec{D}_m)$$

Equation (15) gives insight in the design rule that the quality of the array, given by a combination of resolution, bandwidth and errors, increases when the number of independent points $M$ in the coarray increases.

Equation (15) can be interpreted as a Fourier series to obtain a desired response $RR^*$. When more terms are included a more accurate fit of the desired response is obtained. If various points in the summation over $M$ coincide it will change the weight factors for each frequency accordingly.

The above arguments make also clear that apart from the number of sinusoids $M$ also the two-dimensional distribution of coarray points is of importance. In general a homogeneous distribution of coarray points within a circle with radius $R_c$ will yield a response with a satisfactory trade-off between resolution band width and ripple.

The resolution is $\Delta p$ related to the radius $R_c$ through the approximated expression:

$$\Delta p \approx \frac{1}{R_c}$$

The bandwidth in terms of the Nyquist wavenumber or slowness $p_N$ is given by:

$$p_N = \frac{1}{R_c}$$

in which $r_c$ is the average distance between the coarray points. All of this is well known in standard time series analyses. The ripple is a complicated function of the homogeneity of the
distribution of coarray points. If we disregard the weights \( w_m \) for the moment then it is clear from the above arguments how we should design the coarray, i.e. as a homogeneous set of points within a certain radius.

Focussing on the symmetry of the coarray only two types of symmetric patterns can be used: a square pattern and a hexagonal pattern. Square coarray patterns suffer in general from the fact that for small \( N \) the number of independent pairs of points \( M \) in the coarray will be less than \( \hat{M} \), or a number of points of the coarray will be outside the circle. Hexagonal coarray patterns are easy to construct from array patterns with a three fold symmetry. This class of arrays is called isometric (Haubrich, 1968). Clearly this type of symmetry is most efficient for small \( N \), because of the combination of the plane filling property of hexagonal symmetry and the fact that threefold symmetry will lead to sixfold symmetry in the coarray.

Quite another class of arrays is possible, derived from circular symmetry especially assemblies of uneven numbered polygons are used. The seismic array of Hagfors and the NORESS type arrays are recent examples of polygon arrays. In this type of design the alignment of one ring to the other is more or less arbitrary.

Haubrich distinguishes for linear and two-dimensional arrays between an “optimal” array and a “perfect” array. A perfect array is defined as an array for which \( M = \hat{M} \) and the coarray is uniform. This means for a linear array that the coarray consists of pairs of equidistant points at: \( \pm 1, \pm 2, \pm 3, \ldots, \pm n \). When \( M \) is as large as possible, the array is called optimal. With slight modifications the same definitions can also be applied to the two-dimensional case. The number of perfect linear arrays is limited to \( N \leq 4 \). In the same sense the largest perfect two-dimensional isometric array can be constructed for \( N = 6 \).

We choose this six-element array configuration for our acoustic array illustrated in figure 5a. The coarray is given in figure 5b. The perfect or optimal response however discards the use of weights, or the use of other than classical methods. On the other hand the perfect or optimal arrays are not trivial solutions to the problem.
Operating modes with arrays

The above arguments will lead to an array configuration. Once installed in the field the next step is to operate the array in the best way, considering the type of data and the aim for specific results. In our case, with the perfect six element isometric array, the detector separation was 25 metres. Considering the lowest signal velocity of ca. 300 m/s and a Nyquist wavelength ($\lambda_N$) of ca. 50 m, we can handle frequencies up to 6 Hz. In practice, however, this is a far too strict frequency limitation.

The construction of an analog spatial anti-alias filter is not easy. In most cases, this problem is solved in the time domain by choosing a Nyquist frequency in such a way that given a known dispersion relation wavelengths shorter than $\lambda_N$ are filtered out in the time series of each detector.

Operating the array above the Nyquist wavenumber is possible because of a priori knowledge of the signal in the time domain. For instance, we know that the maximum slowness is ca. 3.3 s/km for sound waves. Secondly, a lot of signals like sonic booms have a broad frequency/wavelength content. Especially the broad frequency/wavelength content of signals make it possible to operate the array at least a factor 5 above the Nyquist wavenumber, the Nyquist frequency was 60 Hz. The background of this paradox lies in the integration over a frequency band by which the lower frequency guide the higher ones. In figure 5e the array response is given over a broad frequency range. This figure shows the increased resolution and the fact that the true main lobe in slowness space is undistorted while the alias lobes are smeared out. In the figures 5c and 5d the array responses are given at 10 Hz.

Given these facts on resolution enhancement with broadband signals, one can relax somewhat on the resolution of the true main lobe and gaining less ripple by applying a taper function on the array for instance, with a two-dimensional Hamming taper. See figure 5d. In the case of the 6-element array the ripple is 13% of the main lobe in the untapered case and 3% in the tapered case, a factor of 4, at the expense of a 20% decrease in resolution. The choice of the tapering function can be optimised through the Backus-Gilbert algorithm which treats the trade-off between errors and resolution in a more formal sense (Backus, 1968).
An alternative way to remove the ripple and the side lobes altogether can be achieved with the procedure CLEAN, originally developed in radio-astronomy by Högbom (Högbom, 1974) and later introduced in seismology by Nolet and Panza (1976). This procedure proceeds in five steps as follows:

a. Find the maximum of the plot at \( p_x, p_y \).

b. Subtract from the normalized \( P \)-plot a certain fraction of the theoretical array response.

c. Store the coordinates of the maximum \( p_x, p_y \) and the fraction.

d. Proceed to point a, if the residual \( P \)-plot contains power exceeding a certain limit else,

e. Sum back over all the obtained \( p_x \) and \( p_y \) values from stage c with a new response of a hypothetical array with one main lobe only.

**Normalization**

When working out the \( f-k \) spectra or \( \omega \)-\( p \)-plots some care should be taken with regard to proper normalization of the Fourier spectra of each detector, e.g. the functions \( S_n(\omega) \). There are three possibilities:

a. No normalization at all. In this procedure the differences in the amplitude factors will enter the summation as weight factors when the magnification of the detectors is not set exactly equal, thereby lowering the resolution.

b. Normalization of \( S_n(\omega) \) with \(|S_n(\omega)|\). This normalization will remove differences in amplitude response, leaving only differences in the phase response. In analysing broadband data with strong frequency dependence within the band, the danger exists that low amplitude frequencies are dominating the \( P \)-plot.

c. To avoid at least in an average sense both above situations, a third normalization scheme is used. It uses the amplitude terms averaged over all detectors.

None of those normalization schemes seems to be ideal, but we tend to favour the third scheme, given reasonable tolerances on amplitudes of the individual detectors to start with. This can be checked by comparing scheme a and c. In narrowband applications scheme b seems appropriate.
Delay and sum method calculation of the best beam

Once the horizontal slowness vector is known, both the horizontal propagation speed and the azimuth of the signal are known. From this the time delay \( \Delta \tau_j \) at each detector can be calculated using:

\[
\Delta \tau_j = -\hat{\beta} \cdot \vec{r}_j
\]  

(18)

So through shifting each trace with the calculated delay to align the individual traces, and then sum or weight and sum the traces, a so-called “best beam” can be calculated. Since the signal is known only at the time intervals of the sampling frequency this delay and sum procedure is done in the frequency domain and not in the time domain. The result; the best beam, gives the best estimate of the signal with noise reduction of a factor of the order of \( \sqrt{N} \) where \( N \) is the number of detectors. Within the same procedure we calculate the integral of the signal, because the output of some detector types represents differentiated pressure. Best beam methods can also be employed in real time event detection (Smart, 1971).

Correlation method

Well known alternatives for the method of f-k analysis as well as for event detection are correlation techniques (Cansi, 1993), (Grachev, 1978). The basic formula for cross-correlation is given by:

\[
c_{g,h}(\tau) = \int_{-\infty}^{\infty} g(t) h(t-\tau) \, dt \quad ; \quad c_{g,h}(\tau) = c_{h,g}(-\tau)
\]  

(19)

The functions \( g(t) \) and \( h(t) \) are the time series of two array elements. The time lag \( \tau \) measures the relative shift of the two channels. The time lag of the absolute maximum of the function \( c_{g,h}(\tau) \) corresponds to the time delay \( \tau_{gh} \) between the signals in the channels \( g \) and \( h \). In some cases the maximum of the correlation function is not quite clear due to the oscillatory motion of the signal or noise components. Other maxima can occur at the characteristic period of oscillation in the case of narrow band signals. A large ambient noise makes proper identification of the maxima difficult. This ambiguity is the correlation counterpart of the Nyquist-problem. The fast way to calculate the correlation function is by the Fast Fourier
Transform (FFT):

\[ C_{g,h}(\omega) = FT(G_{g,h}(\tau)) = G(\omega) H^*(\omega) = A(\omega) \cdot e^{i\phi_c(\omega)} \]  \( \text{(20)} \)

The phase function \( \phi_c(\omega) \) can be interpreted in two equal justified ways. First it can be interpreted as a phase shift in the space-domain. Then it follows that \( \phi_c(\omega) = \frac{2\pi}{k} \cdot \tau_{gh} \) (Grachev, 1978). This leads to standard f-k analysis. Second \( \phi_c(\omega) \) can be interpreted in the time-domain as \( \phi_c(\omega) = \omega \cdot \tau_{gh} \). This leads to the correlation method (Cansi, 1993). The two methods are connected through the dispersion relation \( c = \omega / |k| \). So the methods are in principle fully equivalent. The differences can be found in the ease of calculation and the way intermediate steps can be used to build in to ensure right results. For example, the intermediate rule that \( \sum \phi_c(\omega) = 0 \). This rule is equivalent with the Chasles equations with respect to time delays. Another intermediate calculation can based on the fact that an auto-correlation is symmetric with respect to time lag and has only one maximum when broadband signals are considered.

The time delays \( \tau_{gh} \) are calculated from the slope of the phase as a function of frequency:

\[ \tan(\phi_c(\omega)) = \frac{\text{Im}(C_{g,h}(\omega))}{\text{Re}(C_{g,h}(\omega))} \quad \tau_{gh} = \frac{d\phi_c}{d\omega} \]  \( \text{(21)} \)

Just like the calculation of the group delay in standard time series analysis. The slope of the phase function can be determined by a weighted least-squares fit, after a course estimate of the delay time to avoid the \( 2\pi \) ambiguity. The weighting function can be the coherency function:

\[ |T_{g,h}(\omega)| = \frac{|G(\omega) H^*(\omega)|}{(|G(\omega)|^2 |H(\omega) H^*(\omega)|)^{1/2}} \]  \( \text{(22)} \)

So with the correlation function the time differences of the signals between all the \( N(N-1)/2 \) combinations of array elements can be calculated. The consistency can be checked with the Chasles relation in order to avoid any ambiguity due to narrow band signals:

\[ \tau_y + \tau_{jk} + \tau_{kl} = 0 \]  \( \text{(23)} \)
In some cases a much more simple determination of the time lags is more appropriate, for instance by just determining the maxima of the correlation functions. The expected duration of the signals and the array layout will determine the time window of the time series to be correlated and the maximum time lag respectively for non-moving sources.

When all the delay times are known a fit of the time delays can be made to slowness and azimuth or to other relevant parameters such as the curvature of the wave front. In this analysis non-planar waves are allowed. The correlation method uses the complete frequency band of the signal, but it is not well suited to handle multiple signals.

A natural extension of the correlation method is to calculate, instead of the two-function cross-correlation, a multidimensional cross-correlation function:

$$s_{c_{k,h,i,j,\ldots}}(\tau_1, \tau_2, \tau_3, \ldots) = \int g(t) h(t-\tau_1) i(t-\tau_2) j(t-\tau_3) \ldots dt$$

(24)

This super-correlation function has in principle $N-1$ independent time delays. Therefore in order to limit this number various assumptions can be used to obtain results. When for instance a plane wave is assumed, given the array configuration, the time delays can be calculated given slowness and azimuth or the horizontal wave numbers and the velocity of wave propagation.

Another extension of the method can be found in the fact that the cross-correlations in this application are symmetric around the time lag between the signals of pairs of array elements since the cross-correlations here are essentially auto-correlations of shifted signals buried in noise.

With correlation methods it is easy to transform the incoming channels of an array into time series of signal amplitude, azimuth, horizontal slowness or velocity and the inclination of the signal when the ambient temperature is measured (Baumeler, 1991). This approach is useful for identifying the sources of the infrasonic disturbances with only one array.
The optimal array configuration for the correlation method can differ from an array that is operated with f-k analysis, because in the correlation method more subtle step by step approach can be followed in which checks can be build in every step. This could lead to a less sophisticated array design such as a simple square array layout. In a symmetrical 4-station square array with large distances between array elements in principle one element is redundant. When one element fails, the remaining configurations are all equal. Large distances between array elements can be chosen in accordance with the coherence length of the acoustic signals since spatial aliasing is removed in this method. The conclusion that an array can be larger when broadband signals are considered, then an array design on the basis of the centre band frequency only, can also be drawn in f-k analysis.

**Noise considerations**

When signals with low amplitudes are detected, noise considerations become important. Usually one makes a distinction between the noise of the detection system itself and noise outside the system. The latter can be qualified as a continuous background of unwanted signals; atmospheric pressure noise. In a well-designed system the systems noise should be well below the atmospheric pressure noise. With the current technical means this is easy to achieve.

To measure the noise characteristics we performed a number of separate experiments. By placing two sensors as close as possible together the additive system noise was measured for the two or more collocated sensors. By putting the sensors in a line with various distances between the sensors we could measure the atmospheric noise coherency. The output of the \( j^{th} \) channel, \( s_j(t) \) can be defined as the sum of the signal input \( u_j(t) \) and \( n_j(t) \) the systems own measurement noise. \( u_j(t) \) can be a true signal as well as pressure noise. So:

\[
s_j(t) = u_j(t) + n_j(t)
\]  

(25)

When two or more sensors are collocated then \( u_j(t) = u_k(t) \). The input signals are the same, and the systems noise is uncorrelated:

\[
\langle n_j(t) \cdot n_k(t) \rangle = \delta_{jk}
\]  

(26)
The inner product of $x$ and $y$ is denoted as $\langle x : y \rangle$. $\delta_{jk}$ is the Kronecker delta function (Batties, 1989).

With these correlation functions we can separate the atmospheric pressure noise from the systems noise, leading to a signal to noise ratio. There is however a much simpler method for obtaining virtually the same results. Plotting the individual samples of the two time series $s_j(t)$ and $s_k(t)$ as $x$ and $y$ coordinates one obtains a pixel plot. This is illustrated in figure 6. For the two collocated sensors the two statistical noise distributions are clearly visible; the broad distribution of the coherent atmospheric noise $u(t)$, and the more narrow systems noise distribution $n(t)$ at a right angle. If we neglect for a moment the systems noise, we are left with atmospheric noise and a wanted signal amplitude.

Both atmospheric noise and wanted signal have a spatial coherence. This is represented by the auto-correlation in a certain direction in space. When the raw data is bandpass filtered, the correlation $C_{jk}$ is expressed by:

$$C_{jk} = \frac{\sum_{t=1}^{T} (s_j(t) - \bar{s}_j)(s_k(t) - \bar{s}_k)}{\left(\sum_{t=1}^{T} (s_j(t) - \bar{s}_j)^2\right)^{1/2} \left(\sum_{t=1}^{T} (s_k(t) - \bar{s}_k)^2\right)^{1/2}} \quad (27)$$

Where $s_j$ and $s_k$ are the sample values for the sensors $j$ and $k$. $\bar{s}_j$ and $\bar{s}_k$ are the mean values of $T$ samples. The expression is often referred to as normalized cross-correlation. Proper functioning of the array with low amplitude signals will depend on the short noise coherence length and a long signal coherence length. Coherence length is defined as the width of the auto-correlation as a function of distance between the stations. From the measurement it became clear that for noise a coherence length of smaller than 2 metre is typical for quiet weather. The signal coherence length exceeds 1000 metres for the frequency band of 2-5 Hz.
Examples of array detections

From the Winter of 1991 onwards an acoustic array was operated at De Bilt. From the Summer of 1994 a second but smaller array was operated at Witteveen in the northern part of the Netherlands. The arrays registered a few events per day. Often the detections of the arrays are hard to trace back to certain events, nevertheless, we detected a large number of sonic booms. These detections were from military airplanes and from the France and British Concorde. In certain cases a thunderstorm was detected. The various thunderclaps made it possible to follow the trajectory of the storm. In many cases a number of reflections were recorded especially when the signals originated from the Concorde at distances larger than 1000 km.

Conclusions

In the development of the acoustic infrasonic array we borrowed the technique from seismograph arrays and used perhaps forgotten ideas with respect to the infrasonic detector. In this sense there is nothing new presented here. The purpose of this paper was to show how a low cost acoustic array can be operated with a minimum of effort with respect to data acquisition. By changing the sensors or the dimension of the array a wide variety of physical wave propagation phenomena can be investigated using this technique.
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